Odds : Chance Measurement in Three Contexts

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What are students' views of odds? Students were asked to interpret a newspaper headline, "North at 7-2". Three different perspectives were distinguished: (1) a probability view often using traditional part-whole ratios, (2) a frequency view involving scores and frequency of wins, and (3) a social view, usually involving betting and money exchange in part-part ratios. Each view followed a developmental sequence, with interaction between them.

Odds are indeed odd in relation to traditional probabilities in two respects. The first is that in addition to expressing uncertainty as theoretical ratios or expected relative frequencies, it is possible to express odds in the social context of betting. These three expressions are described by Kapadia (1986) as: (1) the classical probability view, based on symmetry and equal likelihood, (2) the frequency view, based on long term relative frequency, and (3) the subjective view. The subjective view is rarely taught in the schools, except perhaps in relation to odds. *A National Statement on Mathematics for Australian Schools* suggests activities for junior high school students to "understand and explain social uses of chance processes" (Australian Education Council [AEC], 1991, Statement C1, p. 175) including:

- discuss the term 'odds-on' and note that statements of odds which appear in gambling contexts reflect statements of subjective probability as well as statements of return on money invested (e.g., recognise that if the odds are 100 to 1 then the return on a win is very high but the chance of a win is correspondingly small), and
- investigate 'odds' to determine how the bookmaker makes a profit (the related probabilities add up to more than one).

This significant element of social uses of chance processes appears to have been overlooked in *Mathematics - A curriculum profile for Australian schools* (AEC, 1994). Peard (1991, 1994) found experienced gamblers use sophisticated strategies for working with odds which do not use conventional ratio based mathematics. His work suggested a strong involvement of work-place mathematics for handling odds, quite distinct from performance on more traditional school-based probability tasks.

The second respect in which odds are odd is that they involve part-part ratios rather than part-whole ratios; for example the outcomes of a coin toss are expressed as "50-50" rather than "50/100" or "1/2". Singer and Resnick (1992) investigated students' strategies for comparing two boxes with various ratios of coloured marbles, and found that students preferred part-part comparisons rather than part-whole comparisons, concluding "for these problems, at least . . . children's representations are generally based on the parts" (p. 244). If part-part comparisons are preferred, then odds would be preferred to conventional part-whole ratio probabilities, with major implications for the curriculum. There is no evidence yet, however, that Singer and Resnick's conclusion is transferable to odds.

Pilot work by Watson, Collis and Moritz (1994) gave preliminary indications of both context and level of quantification being evident in student responses to a newspaper headline implying odds for a football match. The headline was interpreted by many students at younger levels as a score, suggesting limited exposure to odds. A variety of expressions, including theoretical chance, relative frequency and betting, continued to be used at all levels of the development of appropriate quantification of the reported odds. They proposed a developmental model using the SOLO taxonomy (Biggs & Collis, 1989, 1991; Collis & Biggs, 1991), which was revised in later research related to more general items on likelihood (Watson, Collis & Moritz, 1995).

The SOLO model with multimodal functioning identifies five modes of thought: sensori-motor, ikonic, concrete symbolic, formal and post-formal. The focus in relation to odds is the concrete symbolic (CS) mode, which usually develops across the school years, where symbols are used to represent concrete situations. Within each mode, learning occurs in cycles. Three levels are distinguished within a cycle: unistructural, multistructural, and relational. Each level represents an increasing order of complexity as summarised briefly below. The prestructural level precedes the cycle, in this case most likely representing the ikonic (IK) mode.

(P) *Prestructural responses*, which use none of the elements required to identify the mode in question;

(U) *Unistructural responses*, which use only one relevant aspect of the mode;

(M) *Multistructural responses*, in which several disjoint relevant aspects are processed, usually in sequence;

(R) *Relational responses*, in which an integrated understanding of the relationships between the different aspects is exhibited.

Recent studies have described two U-M-R cycles operating within the CS mode in connection with students' understanding of fractions and decimals (Watson, Collis & Campbell, 1995). These two cycles seem to be associated with (i) the development of a particular concrete concept (U_1-M_1-R_1), usually involving measurement or quantification, followed by (ii) its consolidation and application (U_2-M_2-R_2).

The development of the concept of likelihood was described in two U-M-R cycles by Watson, Collis, & Moritz (1995). In the first U-M-R cycle, U_1 responses recognised uncertainty, M_1 responses qualified the uncertainty, and R_1 responses quantified chance in a single setting, generally as a part-whole ratio, although sometimes as a part-part ratio. The second U-M-R cycle described application of the concept of ratio measurement of chance to problems in multiple settings, such as two boxes of marbles problems for comparing two ratios. In relation to this study, it was hypothesised that a first U-M-R cycle would describe the development of a ratio concept to measure chance, while the second U-M-R cycle would describe applications of this concept to the contexts associated with odds, such as distinguishing between part-part and part-whole ratios, and identifying the direction the odds favour.

Method

A media survey (Watson, 1994; Watson, Collis & Moritz, 1994) using newspaper extracts covering different topics in Chance and Data was administered to 312 Grade 6 students and 393 grade 9 students in Tasmanian government schools during 45 minutes of class time. The survey included the item analysed in this study, shown in Figure 1. Only students who attempted this or later items were included in the analysis.

Subsequent samples of students were surveyed under similar conditions, including 43 grade 7 students and 72 grade 9 students from a private girls' school in South Australia, and 30 grade 8 students and 22 grade 10 students from a coeducational school in the English midlands.

North at 7-2

But we can still win match, says coach

What does "7-2" mean in this headline about the North against South football match? Give as much detail as you can.

From the numbers, who would be expected to win the game?

Figure 1. Item related to odds from media survey about Chance and Data.

Results

Responses generally interpreted the headline within one of three different contexts of expression: probability expressions, frequency expressions and social context expressions. Some students, however, failed to respond or gave irrelevant responses (prestructural for this task), while other responses had aspects of multiple contexts. Within each context of expression, a hierarchy of responses was distinguished; the hierarchy was based on the level of sophistication of the quantification involved. Discussion of each approach follows, with examples of each level within that approach. Tables 1 and 2 provide summary of responses.

Prestructural responses

Some students failed to respond to the task of interpreting the numbers, by (1) non-response, (2) indication that they did not know, or (3) an irrelevant response. The first of the following examples has no relevance to the task. The second response is an example of tautology characteristic of prestructural responses. The last two examples gave some indication of knowing to read "7-2" as "7 to 2" from ikonic experience, but did not frame this within the required CS mode of uncertainty.

P: The weather. [Grade 6]
P: I think just 7-2 and that it's the coach saying to win. [Grade 6]
P-IK: It means like 7 to 2. [Grade 6]
P-IK: 7am - 2pm. [Grade 6]

Probability expressions

Probability expressions involved use of the word 'chance' or 'probability' as the context for interpreting the headline '7-2'. At the unistructural level, responses offered a single idea of uncertainty expressed as chance.

U_1: That's North's chances of winning. [Grade 6]
U_1: It means their chances are 7 to 2. [Grade 6]
U_1: 7-2 means the chance of North winning isn't very high.[Grade 6]

The last response indicated some qualification to the idea of chance, but there was no clear indication that this was an interpretation of the numbers.

Responses at the multistructural level qualified the uncertainty by interpreting the numbers in a primitive quantitative manner.

M_1: 7-2 means that North football team has more chance of winning than South. Because 7 is a larger no. than 2, and 7 stands for North + 2 stands for South, so North has a better chance of winning that South. [Grade 9]
M_1: North has a 7% chance to win and a 2% chance to lose.[Grade 6]
M_1: It means North only has 2 chances of winning whereas South has 7 chances of winning. [Grade 6]
M_1: It means that North has a 7 in 2 chance of beating South. [Gr. 9]

Responses which quantified the chance with a ratio measure were classified at the relational level. Often these responses showed an unconventional measure apparently constructed by the student but which sought to express the chance as a ratio.

R_1: Their chance to win the game, e.g. 2-2 would be an even chance. [Grade 6]
R_1: It means that 7 out of 2 chances of winning the football match, or 3.5 out of 1. [Grade 9]
R_1: That there is, I am not quite sure but I think, 7/10 chance that ~~North~~ [correction] South will win and 2/10 chance that North will win, there is 1/10 chance that there'll be a draw. The odds are stacked against North. [Grade 9]

Responses in the second U-M-R cycle applied the general concept of a quantified measure of chance to the situation of odds. Responses at the U_2 level expressed a consolidated ratio measurement of chance, interpreting '7-2' as a part-whole ratio.

U_2: 7-2 means they have a 2 out of 7 chance to beat South. [Grade 9]

U_2: It means North has a 2/7 chance and the South has 5/7 chance. Basically it means in seven games, South are likely to win five, and North to win two. [Grade 9]

U_2: We are talking about odds I think. The team in question has an estimated chance of 2 in 7. Eg. If you had 7 dice-rolls, then an estimated 2 would come out favourably. [Grade 8]

The last two responses expressed the chance also in the frequency context. These could have been classified at the U_2 level in the frequency approach, although in both cases, the first expression involved chance.

Progression to the M_2 level involved incorporating an additional element specific to odds, namely identification of odds as a part-part ratio, and conversion to an appropriate part-whole ratio.

M_2: 'Experts' believe North has a 7/9 chance of winning and South has 2/9. This means that North is lost likely to win, but South still has a small chance. [Grade 9]

M_2: The odds on North winning are 7-2. If South wins it would be a 2 in 7 chance of it happening. Chance of winning: North 77.77, South 22.22. [Grade 9]

The second response offered a U_2 level expression of part-whole ratio in the second sentence, and then goes on to calculate percentages assuming a part-part ratio. This multiple confused serial response is characteristic of the multistructural level.

Responses which could relate a ratio measurement concept of chance with expression of the appropriate part-part ratio and expression of the direction favoured specific to interpreting odds were classified at the R_2 level.

R_2: North hasn't got a good chance, like a ratio 7:2. A 2/9 chance to win. [Grade 9]

R_2: Out of nine in a scale from one, there is a 7 chance that the other team will win and 2 chance that North will win. [Grade 9]

Frequency expressions

Responses categorised as frequency expressions were those which interpreted the numbers as points in the score of a football game, or as wins and losses in a series of games. Responses which interpreted the numbers as the current score were considered to be frequency expressions which were ikonic prestructural. Although they may be defensible alternative interpretations in another context, they do not recognise the concept of the numbers expressing uncertainty in the intended context.

P-IK: It means 7 goals 2 points. [Grade 6]

P-IK: That North lost by 5 points. [Grade 9]

Responses expressing the numbers as a predicted score were considered unistructural in that they offered a single idea involving uncertainty.

U_1: That is the prediction. [Grade 6]

U_1: The coach thinks they'll win by 7 and the others will only get about 2. [Grade 6]

At the multistructural level, responses qualified uncertainty as a past score which might inform predictions, or as a prediction of a primitive ratio of the scores.

M_1: It means that in their last game they lost 7 goals to 2 goals. But they think there is still a chance. [Grade 9]

M_1: They have a 7-2 chance of winning the match. Every two goals they have, South would probably have 7. [Grade 6]

Both responses above identified uncertainty with the word 'chance', and then qualified this by interpreting the numbers quoted in a frequency expression.

Responses at the relational level integrated their expression of frequency with the newspaper context (which effectively ruled out a current score interpretation) by relating the numbers as quantifying past outcomes of games.

R_1: It means North football team has won two games out of seven. [Grade 7]

R_1: North have lost 5 games against them, but they can win another one, it's not impossible. [Grade 9]

Relational responses expressed the numbers as a frequency of wins as opposed to the points score offered at previous levels, indicating development towards a view of frequency over a longer-term. This development has notably involved the change from a part-part comparison to a part-total comparison which is inappropriate for odds.

The first U-M-R cycle plots the development the concept of quantifying uncertainty as a ratio of long term frequency. The second U-M-R cycle involves the application of this concept to the particular context of odds to include appropriate part-part interpretation of predictive long term frequency in favour of the appropriate team.

At the U_2 level, responses expressed a consolidated ratio concept from the first U-M-R cycle, and included a single idea specific to odds. The first two examples involved a predictive aspect but as a part-whole ratio, while the last example specified the appropriate part-part ratio yet gave no indication that the frequency is used to inform a future uncertain event.

U_2: Out of 3 1/2 games, they would probably win 1. [Grade 9]

U_2: It means if they play 7 games, they'll win two. [Grade 9]

U_2: They have played 9 games and only won 2 and lost 7. [Grade 9]

Responses at the M_2 level included a few ideas specific to odds, such as specifying the appropriate ratio and using this to inform prediction for the future event.

M_2: Out of 9 games North would probably win 7 and South would probably win 2. [Grade 9]

M_2: ~~North has 7 chances.~~ [correction] If they played 9 games, North would win 7 times, South would win 2. [Grade 9]

At the R_2 level, responses integrated a number of ideas specific to odds, including clear indication that the odds favour South.

R_2: It means that for every 9 games they play, they are likely to win 2 out of 9. [South selected as expected to win] [Grade 9]

R_2: These are the odds that North will win, these aren't very good. It means that out of 9 games, North will lose 7 and win 2.[Grade 9]

The second response mentioned "odds" which indicates the betting context, then interpreted the numbers as a long-term expected frequency. Responses at the R_2 level often involved multiple contexts of expression with an overview of the different contexts and a flexibility in which context is expressed.

Social context expressions

Expressions in the social context involved "odds", "votes", "favourites", or money terms. Again at the prestructural level there was an intuitive acknowledgment only of the setting.

P-IK: I think it's a bit like betting on the horses. [Grade 6]

At the unistructural level, responses expressed a single idea involving the role played by the numbers in the headline.

U_1: The number of how many they say are for South football. [Grade 6]

Responses at the multistructural qualified the context, generally specifying terms more clearly, such as identifying '7-2' with the term 'odds'.

M_1: 7-2 means the odds stacked against them. [Grade 6]

M_1: It means that North are 7-2 favourites. [Grade 6]

At the relational level, responses expressed the odds with a ratio measurement description.

R_1: It means that people have given North the odds of 7-2, or 3.5 to 1 chance of winning the match. [Grade 9]

R_1: 7/2 means that the odds are against them. (7-2 is that fraction because fractions aren't used in betting. The real number would be 3.5 - 1.) [Grade 9]

The responses in the first U-M-R cycle identified the social context, and offered interpretations parallel to the frequency and chance contexts of expression in the development of a ratio measure. Responses in the second U-M-R cycle applied this concept, usually to describe exchanges of money. Again development is recognised by the acknowledgment of the appropriate part-part ratio, and by the correct direction favoured. At the U_2 level, responses applied a part-whole ratio interpretation, or an interpretation which did not clearly specify the part-part aspect. Some U_2 responses offered votes interpretations, which included consolidated ratio.

U_2: For every $2 you put in, you get $7 out (if they win). [Grade 9]

U_2: These are the odds of winning. If you put in 2 dollars, you are paid 7 back. That means for every vote, 7 are for North and 2 are for South. [Grade 9]

U_2: It means that 9 people or something were asked who would win the match, and 7 people said North would and 2 people said South would. [Grade 8]

Responses which clearly specified the money transactions for a part-part ratio were classified at the M_2 level. Both examples below illustrate unresolved conflict characteristic of the multistructural level. In the first example, the direction favoured was confused, suggesting the expected winner has an overly good return. In the second example, the direction appears again confused, and the additional frequency part-whole interpretation seems inconsistent with the betting scheme described as a part-part ratio.

M_2: 7-2 means that if you place $2 on the game (probably with bookmakers), you will receive $7 plus your original $2 back if North win. These prices (7-2) are called odds. [Part (b): North expected to win] [Grade 9]

M_2: This terminology is commonly used in gambling. 7-2 means if you put $7 bet on something, you will get your $7 back plus an extra $2. I suppose the heading says for every 7 games they play, they win 2. [Grade 9]

At the R_2 level, responses clearly described the betting situation, specifying the team favoured and the appropriate ratio of money involved.

R_2: 7-2 are the odds or chance the team has of wining. For every 2 dollars you bet, if they win you get $7 back plus your 2 dollars. [Part (b): South more likely to win] [Grade 9]

R_2: It means that the odds are 7/2. For example, if you were betting money on them then you would bet $2.00 and if they won you would get $7.00 back. If you have odds like 2/7 then the bookmakers think that team will win because you have to pay more money than you will get back. [Part (b): South is expected to win because 7-2 isn't very good odds] [Grade 7]

Summary across contexts of expression

Table 1 shows a summary of responses by contexts of expression. The summary description for each SOLO level notes the parallel features found in the different contexts. This parallel nature is often exhibited in responses which offered expressions from multiple contexts.

Table 2 shows the SOLO levels of responses by grade level. The score interpretation of the headline was dominant at younger grade levels. There is a trend for older students to achieve higher levels, as might be expected.

Discussion

Students responded to the interpretation task with three views of probability: a traditional chance expression, a frequency expression, and a social context expression. The social expression involves connotations of gambling, and so it is sometimes avoided by teachers on moral grounds. It is, however, an important part of the curriculum for two reasons. The first is that gambling is so pervasive in Australian

society that it is important to be educated about the mathematics of the social event, as curriculum documents acknowledge (AEC, 1991), and to address moral issues, albeit sensitively. The second reason the social context of probability is important is that it illustrates a subjective view of probability, where people make subjective estimates of the likelihood of outcomes. This view helps students understand that choices and judgements by others based on probability estimates may differ according to prior information, which may be important for later understanding the effect of prior information in studying Bayesian conditional probability.

Teachers need to encourage learning across different contexts, such as building on the R_1 frequency expression of "they have won 2 out of 7 games" to include prediction of uncertainty in the chance context, "they have a 2/9 chance". To do so, teachers need to be aware of the wide range of student understandings about odds, and assist students to build upon their understandings to construct appropriate meanings for odds.

The headline was interpreted as a score by a large number of students, although this tendency appeared reduced with older students in favour of an odds interpretation (see Table 2). The open-ended question did not restrict answers to one context, and readily permitted the defensible score interpretation. These results indicate a baseline of the extent of students' exposure to odds, against which future research may compare to measure the impact of curriculum and social change.

The development of quantification of chance throughout the first U-M-R cycle supports the model of development proposed by Watson, Collis and Moritz (1995), although responses in the current study tended to be at lower levels, probably reflecting students' lack of exposure, or possibly due to the complexities of concepts associated with odds. The second U-M-R cycle highlighted specific features of odds: prediction, the part-part nature of the ratio and the direction favoured. Future research could profitably consider further the issue of part-part comparisons, using the contexts of both this study and that of Singer and Resnick (1992).

Table 1
Overview of levels of response by context of expression

SOLO Level	Summary description	Chance context	Frequency context	Social context
P-IK	No expression of uncertainty or prediction	"7 TO 2"	Score current	Betting
U_1	Simple uncertainty expressed	Chance; Likely	Predicted score	Numbers of how many for South
M_1	Uncertainty qualified or attempted quantification	% chance 7 in 2 chance	For every 2 pts, 7 pts (predictive) Past score predicts	Odds: no clear ratio idea
R_1	Uncertainty quantified in ratio measure	3.5 to 1 chance	Game score: won 2 out of 7	Votes/rating/ratio Odds, ratio clear
U_2	Consolidated ratio as predictor OR part-part ratio	N has 2/7 chance	If play 7, win 2 Played 9, won 2 or 7	Bet \$2, win \$7
M_2	Consolidated ratio as predictor AND part-part ratio	N has 7/9 chance	If play 9, N win 7 N win 7 every 9 games played	Bet \$2, win \$2 + \$7 (North favoured)
R_2	Predictive ratio with correct favoured direction	N has 2/9 chance	If play 9, N win 2	S: Bet \$2, win \$7 + \$2 (South favoured)

Table 2
Response levels by grade

SOLO	Tasmania				South Australia				England			
Level	Grade 6		Grade 9		Grade 7		Grade 9		Grade 8		Grade 10	
-	28	9%	30	8%	0	0%	3	4%	0	0%	3	14%
P	24	8%	2	1%	2	5%	0	0%	0	0%	1	5%
P-IK	149	50%	88	25%	29	67%	31	43%	18	60%	2	9%
U_1	38	13%	46	13%	3	7%	5	7%	1	3%	2	9%
M_1	30	10%	76	21%	2	5%	10	14%	1	3%	9	41%
R_1	8	3%	22	6%	2	5%	8	11%	1	3%	4	18%
U_2	24	8%	62	17%	4	9%	11	15%	9	30%	1	5%
M_2	0	0%	12	3%	0	0%	2	3%	0	0%	0	0%
R_2	0	0%	17	5%	1	2%	2	3%	0	0%	0	0%
Total	301	100%	355	100%	43	100%	72	100%	30	100%	22	100%

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